Clinton N. Y. Sep. 12/66.

Dear Sir_

The solution of your qu. relative to the grazing of the cords, is as follows — r being the radius of that tied at A gives $\frac{1.0472}{2}r^2 = 160$ rods, or $r = \sqrt{160 \div \frac{1.0472}{2}} = \sqrt{152.7979 \times 2} = 12.36\sqrt{2}$ since the sector grazed upon is [illegible] of the area of a semicircle whose radius $= r$. Also r' being the radius of the cord tied at B, we have $\frac{5}{6} \times 1.0472\, r'^2 = 160$ or $r' = 12.36 \times \sqrt{\frac{6 \times 2}{5}}$; & r'' being the radius of the cord tied at C, we have $r'' = 12.36 \times \sqrt{\frac{6}{7}}$, as required.

Your Diophantine question can be solved by using $z+m$, $z+n$, & z for the sought numbers

Then by aſsuming $(z+m)^2 + (z+n)$

$+2z^2=4z^2+2(m+n)z+m^2+n^2=(2z+r)^2$
$=4z^2+4rz+r^2$, we get $z=\frac{m^2+n^2-r^2}{2(2r-m-n)}$
& assuming $(z+m)^2+2(z+n)^2+2z^2=4z^2+$
$2(m+2n)z+m^2+2n^2=(2z+s)^2=4z^2+$
$4sz+s^2$, we get $z=\frac{m^2+2n^2-s^2}{2(2s-m-2n)}$.
Equating the values of z, we
get $\frac{m^2+2n^2-s^2}{2(2s-m-2n)}=\frac{m^2+n^2-r^2}{2(2r-m-n)}$,
which by equating their
numerators & denominators
give $s^2-r^2=n^2$ & $s-r=\frac{n}{2}$, &
thence $s^2-r^2\div(s-r)=2n=s+r$,
& thence $s=\frac{5n}{4}$, $r=\frac{3n}{4}$.
Hence, $z=\frac{m^2+n^2-\frac{9n^2}{16}}{n-2m}=\frac{7n^2+16m^2}{16(n-2m)}$,
$z+m=\frac{7n^2+16mn-16m^2}{16(n-2m)}$, & $z+n=$
$\frac{23n^2-32mn+16m^2}{16(n-2m)}$; & rejecting
the useless denominators, we may
clearly take $7n^2+16m^2$, $7n^2+16mn$
$-16m^2$, $23n^2-32mn+16m^2$, for
z, $z+m$, & $z+n$.
Hence, the remaining condition of the question gives
$z^2+(z+n)^2+2(z+m)^2=(7n^2+16m^2)^2+$
$(23n^2-32mn+16m^2)^2+2(7n^2+16mn-16m^2)^2$
$=676n^4-1024n^3m+2048n^2m^2-2048nm^3+1024m^4$
$=4(169n^4-256n^3m+512n^2m^2-512nm^3$
$+256m^4)^2=$ a square; consequently since $4=2^2$ a square, we
must make $169n^4-256n^3m+$
$512n^2m^2-512nm^3+256m^4$
equal to a square.

Hence, by assuming $169n^4-$
$256n^3m+512n^2m^2-512nm^3+256m^4$
$=(13n^2-16nm+16m^2)^2=169n^4-416n^3m$
$+672n^2m^2-512nm^3+256m^4$ or
by an obvious reduction, we
shall have $-256n^3m+512n^2m^2$
$=-416n^3m+672n^2m^2$, or
$672n=160m$ or $\frac{21}{5}=\frac{m}{n}$,
which is satisfied by
putting $m=21$ & $n=5$.
Putting 21 & 5 for m & n
in the values $7n^2+16m^2$, $7n^2+$
$16mn-16m^2$, & the values of
&c, we shall have
z, $z+m$, ~~1155~~, & 4271, are the values
7231, -5201, & $z+n$
of z, $z+m$,

Prof Strong

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